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Investigation of Some Fuzzy Optimization Problems with Fuzzy Genetic Algorithms

Abbas Akrami

Department of Mathematics, Faculty of Science, University of Zabol, Zabol, Iran

Corresponding author's e-mail: *akrami.ab@uoz.ac.ir* and *akramiabas@yahoo.com*

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Abstract

Fuzzy optimization techniques have proven to be highly effective in the field of optimization, particularly in scenarios where decision-making processes are complex and influenced by uncertainty. These methods address vagueness and ambiguity by leveraging the principles of fuzzy logic, making them applicable across various domains such as economics, engineering, healthcare, and environmental management. Optimization techniques are essential for enhancing performance and efficiency in numerous industries. Among these, fuzzy logic provides a robust framework for handling uncertainties and imprecision commonly encountered in real-world problems. In this paper, we explore fuzzy genetic algorithms as a solution to certain fuzzy optimization problems. We demonstrate that this approach yields a reliable approximation of solutions for such problems. Additionally, we illustrate the application of this algorithm in three key areas: maximum fuzzy flow, fuzzy regression, and fuzzy controller design. The foundation of fuzzy genetic algorithms lies in the discretization of interval-based fuzzy subsets. These algorithms offer an innovative way to generate approximate solutions for fuzzy optimization problems where variables are arbitrary fuzzy subsets of specific intervals. This makes them versatile and applicable to a wide range of challenges.

1. Introduction

Optimization techniques play a critical role in enhancing efficiency and performance across various fields, including engineering, economics, healthcare, and environmental management. However, real-world optimization problems are often characterized by uncertainty, imprecision, and vagueness, making traditional methods less effective. Fuzzy optimization has emerged as a powerful approach to address these challenges by incorporating fuzzy logic, which allows for the representation and manipulation of uncertain or ambiguous data. By extending classical optimization methods, fuzzy optimization provides robust solutions to problems where parameters or constraints are not precisely defined [1].

Genetic algorithms (GAs), inspired by biological evolution and natural selection, have gained significant attention as a stochastic search method for solving complex optimization problems. These algorithms mimic processes such as inheritance, mutation, and selection to iteratively evolve solutions toward optimality [2]. GAs are particularly effective in handling non-linear, multi-modal, and high-dimensional problems, making them a popular choice in regression-based prediction, pattern recognition, and system modeling [3]. When combined with fuzzy logic, GAs can effectively tackle optimization problems involving uncertainty, leading to the development of fuzzy genetic algorithms (FGAs) [4, 5].

Recent advancements in fuzzy optimization and GAs have demonstrated their applicability in diverse domains. For instance, in [6], it was proposed an adaptive FGA for multi-objective optimization in engineering design, showcasing its ability to handle complex, real-world constraints. In [7], the authors introduced a hybrid fuzzy-GA approach for optimizing supply chain networks under uncertain demand, highlighting its efficiency in decision-making under uncertainty. Additionally, Zhang and Li [8] explored the use of FGAs in energy management systems, demonstrating their effectiveness in optimizing energy consumption with imprecise data. Furthermore, Wang and Chen [9] applied fuzzy chaos simulation to solve complex optimization problems, while Kumar and Singh [10] and Buckley and Hayashi [11] utilized triangular and trapezoidal fuzzy numbers to address fuzzy optimization problems with crisp variables.

In this paper, we present FGA designed to solve fuzzy optimization problems. The proposed method leverages the discretization of interval-based fuzzy subsets and integrates fuzzy logic with GAs to generate approximate solutions for problems where variables are represented as arbitrary fuzzy subsets of specific intervals [12]. We demonstrate the application of this algorithm in several key areas, including maximum fuzzy flow, fuzzy regression, and fuzzy controller design. The results illustrate the algorithm's ability to provide efficient and reliable solutions to complex optimization problems under uncertainty.

2. Basic Concepts

In this section, we introduce the definitions and symbols used in the article. Consider the function F defined as follows ([8]):

$$\tilde{Y} = F(\tilde{X}),$$

which $\tilde{X}$ is a fuzzy subset of the interval $[0, U], U > 0$. Thus F is a well-defined mapping from fuzzy subsets of $[0, U]$ to fuzzy subsets of real numbers. The goal is to find $\tilde{X}^*$, the maximum point of the function in the interval $[0, U]$. Since $\tilde{Y}$ is a fuzzy set, its maximum cannot be directly determined. To address this, we introduce a measure function μ that maps each fuzzy set to a real number:

$$\mu(\tilde{Y}) = \eta.$$

Our objective is to find $\tilde{X}^*$ such that η is maximized. To apply the FGA, we first discretize the fuzzy set $\tilde{X}$. Let N be a positive integer. Define $v_0 = 0$ and $v_i = i.\frac{U}{N}, i = 1,2,\dots,N$. Then $\tilde{X}$ can be represented as

$$\tilde{X} = (x_0, x_1, \dots, x_N),$$

where $x_i = X(v_i), i = 0,1,\dots,N$. By applying the discretization of $\tilde{X}$ to F, we obtain $\mu(F(x_0, x_1, \dots, x_N))$. Consequently, F becomes a mapping from $[0,1]^{N+1}$ to set of all real numbers. Our task is to find a vector $\sigma \in [0,1]^{N+1}$ that maximized η.

2.1 *FGA*

To find $\sigma \in [0,1]^{N+1}$ that maximized η, we implement the following steps:

1. Initial Population Generation:
    - Randomly generate an initial population of size n from $[0,1]^{N+1}$.
    - Represent the population as

$$\tilde{X}_1 = (x_{10}, x_{11}, \dots, x_{1N}),$$
$$\vdots$$
$$\tilde{X}_n = (x_{n0}, x_{n1}, \dots, x_{nN}),$$

where $x_{ij} \in [0,1]$.

2. Fitness Evaluation:
    - Calculate η_i for each $\tilde{X}_i$, $(i = 1, \dots, n)$.

    Let $S = \eta_1 + \eta_2 + \dots + \eta_n$, and define $S_k = \eta_1 + \eta_2 + \dots + \eta_k$ for $k = 1,2, \dots, n$.
    - Construct intervals I_i as follows:

$$I_1 = [0, S_1),$$
$$I_i = [S_{i-1}, S_i) \text{ for } i = 2, \dots, n,$$
$$I_n = [S_{n-1}, S_n].$$

3. New Population Generations:
    - For each i $(1 \le i \le n)$ generate a random number $w_i \in [0, S]$.
    - If $w_i \in I_i$, then select $\tilde{X}_i$ to be part of the new population P_i.
4. Crossover.
    - Perform crossover on pairs (P_1, P_2), (P_3, P_4), etc.
    - Let $(0 \le p \le 1)$ be the probability of crossover.
    - For each pair (P_1, P_2), generate a random number $x \in [0,1]$.
    - If $x \le p$, then perform crossover. Otherwise, P_1 and P_2 remain unchanged.
    - If crossover is performed, then generate two children:

$$P_1' = (p_{10}, p_{11}, p_{12}, p_{13}, \odot, \odot, \dots),$$
$$P_2' = (p_{20}, p_{21}, p_{22}, p_{23}, *, *, \dots).$$

    - Repeat this process for all pairs (P_i, P_{i+1}) to create new population $P_1{}', P_2, \cdots$.
5. Mutation:
    - Let q $(0 \le q \le 1)$ be the probability of mutation.
    - For each child $P_i{}'$, generate a random number $w_i \in [0,1]$.
    - If $w_i \le q$, then mutate $P_i{}'$ by chancing one of its positions.
    - After mutation, identify $\tilde{X}_i$ with $P_i{}'$ and proceed to the next iteration.
6. Termination:
    - Repeat steps 2 to 5 for M iterations, where M is the maximum number repetitions.
    - The maximum value of η and the corresponding $\tilde{X}$ are determined by the GA.

3. Applications of GAs

In this section, we describe three applications of FGA.

1. Fuzzy maximum flow problem [9].

Let $\tilde{C}_{ij}$ be non-negative triangular fuzzy numbers that show the maximum flow capacity on an arc from node $i$to node j. Let $\tilde{X}_{ij}$denote the fuzzy flow from node i to node j, such that $\tilde{X}_{ij} \leq \tilde{C}_{ij}$. Suppose that $U_1 > 0$ is chosen such that $\tilde{X}_{13} \subseteq [0, U_1]$, which implies $\tilde{X}_{13} \leq \tilde{C}_{13}$. Similarly, select $U_2 > 0$ such that that $\tilde{X}_{12} \subseteq [0, U_2]$, ensuring $\tilde{X}_{12} \leq \tilde{C}_{12}$ and $\tilde{X}_{12} \leq \tilde{C}_{24}$. Finally, we choose $U_3 > 0$ such that $\tilde{X}_{35} \subseteq [0, U_3]$, implies $\tilde{X}_{35} \leq \tilde{C}_{35}$ and $\tilde{X}_{35} \leq \tilde{C}_{56}$.

The mathematical formulation of the problem is as follows:

$$
\begin{aligned}
& max\,\tilde{X} \\
& s.t. \\
& \tilde{X} = \tilde{X}_{12} + \tilde{X}_{13}, \\
& \tilde{X}_{24} = \tilde{X}_{12}, \\
& \tilde{X}_{13} = \tilde{X}_{34} + \tilde{X}_{35}, \\
& \tilde{X}_{46} = \tilde{X}_{24} + \tilde{X}_{34}, \\
& \tilde{X}_{56} = \tilde{X}_{35}, \\
& \tilde{X} = \tilde{X}_{46} + \tilde{X}_{56}, \\
& 0 \leq \tilde{X}_{ij} \leq \tilde{C}_{ij}.
\end{aligned}
$$

To handle constraints, we introduce penalty terms as follows:

$$
\begin{aligned}
& \tilde{\Omega}_1 = \tilde{C}_{34} - (\tilde{X}_{13} - \tilde{X}_{35}), \\
& \tilde{\Omega}_2 = \tilde{C}_{46} - (\tilde{X}_{12} + \tilde{X}_{13} - \tilde{X}_{35}), \\
& \tilde{\Omega}_3 = \tilde{X}_{13} - \tilde{X}_{35}.
\end{aligned}
$$

The problem is then reformulated as:

$$max(\tilde{X} + \Psi_1\tilde{\Omega}_1 + \Psi_2\tilde{\Omega}_2 + \Psi_3\tilde{\Omega}_3),$$

where $\tilde{\Omega}_1, \tilde{\Omega}_2, \tilde{\Omega}_3$ are penalty coefficients. We use the FGA to solve this fuzzy optimization problem.

2. Fuzzy regression.

Consider a system identification problem, where the input data $\tilde{X}_i$ and output $\tilde{Y}_i$ are known, but the exact structure of H is unknown. We have:

$$\tilde{Y}_i = H(\tilde{X}_i), i = 1,2, \dots, n.$$

We assume a linear relationship between the input and output:

$$\tilde{Y} = \tilde{A}\tilde{X} + \tilde{B},$$

where $\tilde{A}, \tilde{B}$are unknown fuzzy sets. The goal is to fit this linear equation to the data, a process known as fuzzy linear regression. We aim to find $\tilde{A}$, $\tilde{B}$ such that the error term is minimized. Let:

$$\tilde{W}_i = \tilde{A}\tilde{X}_i + \tilde{B}, \qquad i = 1,2, \dots, n.$$

If $d(\tilde{Y}_i, \tilde{W}_i)$ represent a distance measure between $\tilde{W}_i$, $\tilde{Y}_i$, then the total error is:

$$E = \sum_{i=1}^{n} d(\tilde{Y}_i, \tilde{W}_i).$$

Since the FGA is designed for maximization, we transform the problem by introducing a new variable η:

$$\eta = M - E, M > 0.$$

We discretize the intervals $\tilde{A} \subseteq [0, M_1], M_1 > 0$ and $\tilde{B} \subseteq [-M_2, M_2], M_2 > 0$, we discretize the intervals $[-M_2, M_2]$ and $[0, M_1]$ and apply the FGA to maximize η.

4. Conclusion

In this paper, we proposed (an FGA) to solve fuzzy optimization problems. The algorithm leverages the discretization of interval-based fuzzy subsets and combines the principles of fuzzy logic with GAs to handle uncertainty and imprecision effectively. We demonstrated the application of this algorithm in two important examples, including the maximum fuzzy flow problem and fuzzy regression.

The key contributions of this work are as follows:

1. Fuzzy Maximum Flow Problem: We applied the FGA to determine optimal fuzzy flows in a network with fuzzy capacities, ensuring that the constraints are satisfied while maximizing the total flow.
2. Fuzzy Regression: We used the FGA to fit a linear fuzzy model to data with uncertain input-output relationships, minimizing the error between predicted and observed values.
3. General Applicability: The proposed algorithm is versatile and can be adapted to a wide range of fuzzy optimization problems where variables are represented as arbitrary fuzzy subsets of specific intervals.

The results highlight the effectiveness of the FGA in generating approximate solutions to complex optimization problems under uncertainty. By discretizing fuzzy subsets and incorporating penalty terms for constraints, the algorithm provides a robust framework for solving real-world problems with imprecise or incomplete data.

Future work could explore the application of this algorithm to other domains, such as fuzzy control systems, fuzzy scheduling and fuzzy decision-making. Additionally, further improvements could be made to enhance the algorithm's efficiency and scalability for large-scale problems.

In summary, the FGA presented in this paper offers a powerful and flexible approach to solving fuzzy optimization problems, making it a valuable tool for researchers and practitioners in various fields.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding this article.

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