

# Dynamic Characterization of One-Dimensional Consolidation: A Rationalized Haar Wavelet Transform Approach for Variable Coefficient of Consolidation

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## Abstract

The classical theory of one-dimensional consolidation, originally developed by Terzaghi, is based on the simplifying assumption that the coefficient of consolidation ( $C_v$ ) remains constant throughout the dissipation of excess pore water pressure. In practical geotechnical engineering applications, however, saturated cohesive soils frequently exhibit permeability and compressibility that vary with both time and depth, leading to a coefficient of consolidation that is inherently non-uniform. To address this limitation, the present study introduces an analytical framework that incorporates the Rationalized Haar Wavelet Transform for solving the one-dimensional consolidation equation with variable ( $C_v$ ). Two functional forms are proposed to represent ( $C_v$ ) as explicit functions of time and space, enabling a more realistic characterization of soil behavior during consolidation. The results demonstrate that accounting for variable ( $C_v$ ) produces higher excess pore pressure predictions compared with the classical constant-parameter solution, thereby increasing the estimated duration required to achieve full consolidation. This outcome indicates that traditional analyses may underestimate consolidation times in engineering design. Furthermore, the proposed methodology offers substantial flexibility by allowing a wide range of explicit ( $C_v$ ) functions to be incorporated, making it adaptable to diverse soil conditions and site-specific characterization. The study underscores the significance of incorporating variable soil properties in consolidation analysis and provides a robust computational approach for advanced geotechnical modeling.

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## 1. Introduction

Terzaghi's pioneering theory of one-dimensional consolidation [1] has served as the foundation of modern geotechnical engineering. However, its simplifying assumptions—particularly the use of constant soil parameters—limit its applicability to real-world problems. In natural clay deposits, both the coefficient of permeability and the coefficient of volume compressibility evolve during loading, resulting in a coefficient of consolidation ( $C_v$ ) that varies with depth and time [2-6]. These variations arise from changes in effective stress, fabric rearrangement, and nonlinear compressibility of clay minerals.

To address these complexities, numerous researchers have developed nonlinear consolidation theories and advanced numerical or semi-analytical methods. Gibson et al. [3] introduced finite-strain consolidation theory, while Ying-chun et al. [4] and Lekha et al. [5] incorporated variable compressibility and permeability into the governing equations. More recent studies have explored non-Darcian flow, large-strain effects, and coupled hydro-mechanical behavior [6-9]. Comprehensive reviews of nonlinear consolidation models are provided in [10, 11].

A key challenge in nonlinear consolidation analysis is the selection of an efficient and accurate solution technique. Wavelet-based methods, particularly the Haar wavelet, have gained attention due to their simplicity, sparse matrix structure, and suitability for boundary-value problems. Several authors have successfully applied Haar wavelets to differential and integral equations [12-16]. The Rationalized Haar Wavelet Transform (RHWT), in particular, offers enhanced accuracy and computational efficiency.

In this study, the RHWT is employed to solve the nonlinear consolidation equation with variable ( $C_v$ ). Two functional forms are considered: (i) variation with depth based on penetration test data [17] as illustrated in Figure 1, and (ii) variation with time based on experimental observations [18] as shown in Figure 2. The objective is to quantify the influence of variable ( $C_v$ ) on pore pressure dissipation and consolidation time, and to demonstrate the capability of RHWT in handling nonlinear geotechnical problems.

## 2. $C_v$ Formulation

House et al. [17] proposed a depth-dependent relationship for ( $C_v$ ), shown in Figure 1. A best-fit curve was obtained using MATLAB curve-fitting tools:

$$C_v(z) = -0.0025z^2 + 0.1928z + 1.3044 \quad [R^2 = 0.99], \quad (1)$$

where  $z$  is the depth of the interested point in meter and  $C_v$  is in  $\text{m}^2/\text{year}$ .

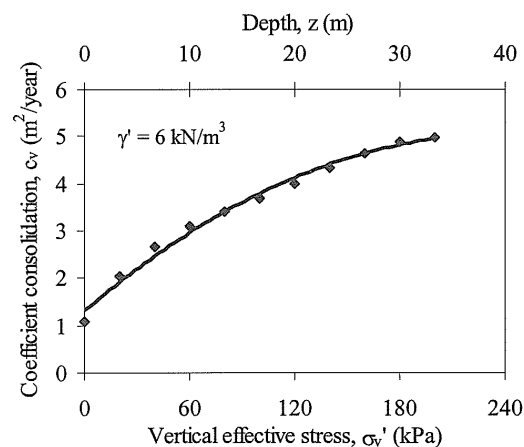
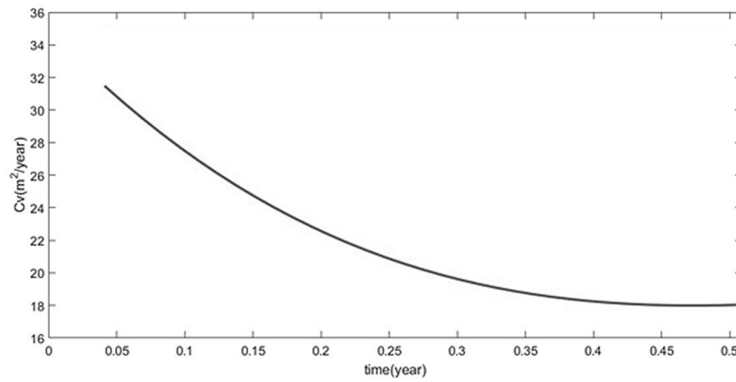


Figure 1. Coefficient of consolidation vs depth [17]



**Figure 2. Variation of  $C_v$  with time [18]**

Similarly, Abbasi et al. [18] demonstrated that ( $C_v$ ) decreases with time during consolidation. A suitable mathematical expression based on Figure 2 is

$$C_v(t) = 17.34 \exp(-7.09 t) + 18.38, \quad (2)$$

where  $t$  is in year and  $C_v$  is in  $m^2/year$ .

### 3. Numerical Solution

The governing equation for one-dimensional consolidation is [19]

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}, \quad (3)$$

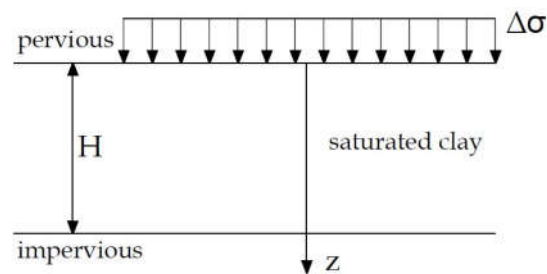
where  $u(z, t)$  is the excess pore water pressure at any vertical depth  $z$  and time  $t$  and  $C_v$  is the coefficient of consolidation of soil which is considered a constant scalar throughout the consolidation process.

Substituting (1) and (2) into (3) yields

$$\frac{\partial u}{\partial t} = C_v(z) \frac{\partial^2 u}{\partial z^2}, \quad (4.a)$$

$$\frac{\partial u}{\partial t} = C_v(t) \frac{\partial^2 u}{\partial z^2}. \quad (4.b)$$

In the case where a cohesive soil layer of thickness  $H$  located below the groundwater level and between two layers of permeable and impermeable soil layers, if a surcharge of intensity  $\Delta\sigma$  is applied at the ground surface over a very large area as shown in Figure 3, the initial and boundary conditions would be as follows:



**Figure 3. Cohesive soil layer and its boundary conditions**

Initial condition:  $u(z, 0) = \Delta\sigma$  and boundary conditions:  $u(0, t) = 0$  and  $\partial u(H, t) / \partial z = 0$  for  $t > 0$ . In this paper, the Haar wavelet method is developed for solving (4) with the aforementioned initial and boundary conditions. Many authors have investigated the use of the Haar wavelet in order to solve differential and integral equations such as Aziz & Amin [12], Shah *et al.* [13], Shiralashetti & Deshi [14] and Rastegar *et al.* [15]. This method exhibits several advantageous features as follows: a) Integrating different functions for coefficient of consolidation. b) High accuracy, fast transformation and possibility of implementation of fast algorithms compared with other known methods. c) Simplicity and small computation costs, resulting from the sparsity of the transforms matrices and the small number of significant wavelet coefficients. d) Convenience of use for solving boundary value problems, since the boundary conditions are taken care of automatically.

In order to solve the problem, the highest derivative in (4) is expanded into RH series as follows:

$$\dot{u}''(z, t) = \sum_{i=1}^{m-1} c_i h_i(z) = \mathbf{C}^T \mathbf{H}(z), \quad t \in (t_s, t_{s+1}), \tag{5}$$

where  $m = 2^j$  and  $h_n(t)$  are RH functions for any  $n = 1, 2, \dots$ , where  $n = 2^j + k$  for  $j = 0, 1, \dots$  and  $k = 0, 1, \dots, 2^j - 1$ . Also,  $h_0(t) = 1$  is defined for all  $t \in [0, 1]$ . Here the integer  $2^j, j = 0, 1, \dots$ , indicates the level of the wavelet,  $k = 0, 1, \dots, 2^j - 1$  is the translation parameter and  $c_i$ 's are the RH function coefficients.

Integration of (5) once with respect to  $t$  from  $t_s$  to  $t$  leads to

$$u''(z, t) = (t - t_s) \mathbf{C}^T \mathbf{H}(z) + u''(z, t_s) \tag{6}$$

and after integrating it twice with respect to  $z$ , from 0 to  $z$ , we have

$$u(z, t) = (t - t_s) \mathbf{C}^T \mathbf{Q} \mathbf{H}(z) + u(z, t_s) - u(0, t_s) + z(u'(0, t) - u'(0, t_s)) + u(0, t). \tag{7}$$

After taking the derivative of (7) once with respect to  $t$ , we obtain

$$\dot{u}(z, t) = \mathbf{C}^T \mathbf{Q} \mathbf{H}(z) + z \dot{u}'(0, t) + \dot{u}(0, t). \tag{8}$$

With using (7) and (8) in  $z = 1$  and the initial and boundary conditions, we have

$$u'(0, t) - u'(0, t_s) = -(t - t_s)^2 \mathbf{C}^T \mathbf{Q} \mathbf{H}(z) - u(0, t) + u(0, t_s) + u(1, t) - u(1, t_s) \tag{9}$$

and

$$u'(0, t) = \dot{u}(1, t) - \mathbf{C}^T \mathbf{Q} \mathbf{H}(z) f - \dot{u}(0, t), \tag{10}$$

where the vector  $f$  is defined as  $f = \begin{bmatrix} 1, 0, \dots, 0 \\ (m-1) \text{ elements} \end{bmatrix}$ . With substitute (9) and (10) into (6)-(8) and assume

$t \rightarrow t_{s+1}$  and  $z \rightarrow z_i$  we have

$$u''(z_i, t_{s+1}) = (t_{s+1} - t_s) \mathbf{C}^T \mathbf{H}(z) + u''(z_i, t_s) \tag{11}$$

and

$$u(z_i, t_{s+1}) = (t_{s+1} - t_s) \mathbf{C}^T \mathbf{Q} \mathbf{H}(z_i) + u(z_i, t_s) - u(0, t_s) + u(0, t_{s+1}) - z_i \left[ (t_{s+1} - t_s) \mathbf{C}^T \mathbf{P} f + u(1, t_s) - u(1, t_{s+1}) + u(0, t_{s+1}) - u(0, t_s) \right] \tag{12}$$

and

$$\dot{u}(z_i, t_{s+1}) = \mathbf{C}^T \mathbf{Q} \mathbf{H}(z_i) + \dot{u}(0, t_{s+1}) - z_i \left[ \mathbf{C}^T \mathbf{P} f - \dot{u}(1, t_{s+1}) + \dot{u}(0, t_{s+1}) \right] \tag{13}$$

Substituting (1), (2), (11)-(13) into (4) in  $(z_i, t_{s+1})$ , we have

$$\begin{aligned} \mathbf{C}^T \mathbf{QH}(z_i) + \dot{u}(0, t_{s+1}) - z_i \left[ \mathbf{C}^T \mathbf{Pf} - \dot{u}(1, t_{s+1}) + \dot{u}(0, t_{s+1}) \right] = \\ [-0.0025z_i^2 0.1928z_i + 1.3044] u''(z_i, t_{s+1}) \end{aligned} \quad (14.a)$$

$$\begin{aligned} \mathbf{C}^T \mathbf{QH}(z_i) + \dot{u}(0, t_{s+1}) - z_i \left[ \mathbf{C}^T \mathbf{Pf} - \dot{u}(1, t_{s+1}) + \dot{u}(0, t_{s+1}) \right] = \\ [17.34 e^{-7.09t_{s+1}} + 18.38] u''(z_i, t_{s+1}) \end{aligned} \quad (14.b)$$

From these equations, the wavelet coefficients  $\mathbf{C}^T$  can be successively calculated and the excess pore water pressure for any location and time can be obtained from (12) subsequently.

## 4. Numerical Results

A double-layered soil profile model is considered for the analysis which is presented in Figure 3. The top layer is freely drained whereas the bottom layer is impervious (one-way drainage). Two cases were considered relating to (14.a) and (14.b) respectively and the results were compared with constant  $C_v$  solution based on [15].

### 4.1 Variable $C_v$ with depth

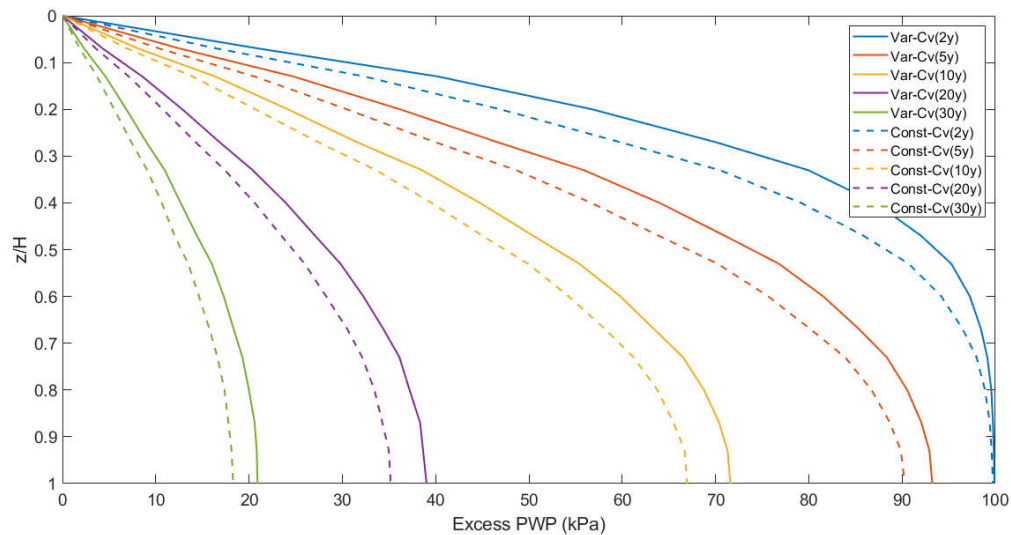
In this case, the boundary condition of the soil layer is taken as one-way drainage and  $C_v$  variations are considered according to (4.a). The excess pore water pressure values under a sustained load of 100 kPa are calculated and presented in Table 1. Another analysis has been conducted with constant  $C_v$  equal to  $2.18 (m^2 / year)$  (which is the mean  $C_v$  over the entire depth of the consolidated layer) and presented in Table 2. Figure 4 shows a graphical view of the data shown in Tables 1 and 2. As can be seen in Figure 4, dissipation of excess pore water pressure is slower in case of variable  $C_v$  which leads to longer duration for complete consolidation.

**Table 1. Excess pwp (kPa) values for the case of variable  $C_v$  with depth**

$z$	$t=0$	2 year	5 year	10 year	20 year	30 year
0	100	0	0	0	0	0
(1/15)H	100	21	12.55	8.1	4.31	2.32
(2/15)H	100	40.28	24.77	16.4	8.57	4.62
(3/15)H	100	56.77	36.18	24.17	12.85	6.85
(4/15)H	100	69.98	46.59	31.54	16.83	8.99
(5/15)H	100	79.93	55.88	38.52	20.39	10.97
(6/15)H	100	87.14	64	44.75	23.94	12.67
(7/15)H	100	92.01	70.97	50.42	27.07	14.41
(8/15)H	100	95.27	76.78	55.4	29.8	16.01
(9/15)H	100	97.27	81.56	59.81	32.26	17.29
(10/15)H	100	98.46	85.39	63.42	34.41	18.36
(11/15)H	100	99.14	88.36	66.52	36.12	19.29
(12/15)H	100	99.57	90.55	68.77	37.21	19.99
(13/15)H	100	99.74	92.07	70.39	38.34	20.6
(14/15)H	100	99.84	92.93	71.3	38.66	20.83
H	100	99.9	93.24	71.59	39.03	20.91

**Table 2. Excess pwp (kPa) values for the case of constant  $C_v = 2.18 (m^2 / year)$**

$z$	$t=0$	2 year	5 year	10 year	20 year	30 year
0	100	0	0	0	0	0
(1/15)H	100	16.52	10.35	6.92	3.57	1.86
(2/15)H	100	32.4	20.62	13.97	7.12	3.71
(3/15)H	100	46.92	30.48	20.69	10.76	5.54
(4/15)H	100	59.72	39.91	27.38	14.26	7.38
(5/15)H	100	70.44	48.64	33.59	17.41	9.09
(6/15)H	100	79.17	56.68	39.56	20.6	10.65
(7/15)H	100	85.84	63.89	44.98	23.45	12.01
(8/15)H	100	90.71	70.22	49.9	25.96	13.47
(9/15)H	100	94.21	75.66	54.27	28.27	14.64
(10/15)H	100	96.5	80.19	58.17	30.54	15.76
(11/15)H	100	97.98	83.89	61.22	32.13	16.62
(12/15)H	100	98.89	86.73	63.78	33.44	17.37
(13/15)H	100	99.4	88.7	65.51	34.3	17.79
(14/15)H	100	99.6	89.87	66.7	34.99	18.13
H	100	99.74	90.29	66.96	35.19	18.3



**Figure 4. Results comparison for case 1**

**4.2 Variable  $C_v$  with time**

Similar to previous case, the numerical solution has been conducted considering the variability of  $C_v$  with time. The governing equation for coefficient of consolidation is (2) which has an exponential mathematical form. The excess pore water pressure values under a sustained load of 100 kPa are calculated and presented in Table 3. For comparison with classical consolidation case, a constant  $C_v$  equal to  $35.72 (m^2 / year)$  (which is the mean  $C_v$  up to 30 year consolidation duration) is chosen and the excess pore water pressures are calculated based on [15] which is presented in Table 4. Figure 5 shows a side-by-side comparison of the proposed method with the

classical method. As it is shown clearly in Figure 5, the differences between variable and constant cases are significant and because in reality the  $C_v$  decreases over time, the excess pore water pressure dissipates slower than the case of constant  $C_v$  which leads to longer complete consolidation process.

**Table 3. Excess pwp (kPa) values for the case of variable  $C_v$  with time**

$z$	$t=0$	1 year	2 year	3 year	4 year
0	100	0	0	0	0
(1/15) $H$	100	8.09	5	3.11	1.96
(2/15) $H$	100	16.08	10.02	6.28	3.95
(3/15) $H$	100	23.92	14.88	9.28	5.77
(4/15) $H$	100	31.46	19.67	12.35	7.59
(5/15) $H$	100	38.58	24.17	15.13	9.46
(6/15) $H$	100	45.27	28.47	17.84	11.24
(7/15) $H$	100	51.43	32.41	20.37	12.79
(8/15) $H$	100	57.05	36	22.74	14.22
(9/15) $H$	100	61.91	39.18	24.72	15.49
(10/15) $H$	100	66.17	42.03	26.48	16.65
(11/15) $H$	100	69.65	44.33	28	17.64
(12/15) $H$	100	72.43	46.14	29.15	18.28
(13/15) $H$	100	74.4	47.45	29.99	18.84
(14/15) $H$	100	75.58	48.23	30.42	19.14
$H$	100	76.05	48.46	30.56	19.24

**Table 4. Excess pwp (kPa) values for the case of constant  $C_v = 35.72 (m^2 / year)$**

$z$	$t=0$	1 year	2 year	3 year	4 year
0	100	0	0	0	0
(1/15) $H$	100	5.59	2.38	0.97	0.41
(2/15) $H$	100	11.22	4.69	1.95	0.82
(3/15) $H$	100	16.65	6.93	2.87	1.22
(4/15) $H$	100	21.89	9.19	3.88	1.6
(5/15) $H$	100	26.94	11.36	4.78	1.96
(6/15) $H$	100	31.63	13.3	5.45	2.31
(7/15) $H$	100	36.11	15.16	6.22	2.61
(8/15) $H$	100	40.05	16.86	6.91	2.91
(9/15) $H$	100	43.62	18.4	7.62	3.16
(10/15) $H$	100	46.63	19.65	8.07	3.41
(11/15) $H$	100	49.22	20.75	8.59	3.57
(12/15) $H$	100	51.25	21.65	8.91	3.74
(13/15) $H$	100	52.7	22.25	9.17	3.84
(14/15) $H$	100	53.64	22.67	9.4	3.91
$H$	100	53.89	22.79	9.37	3.92

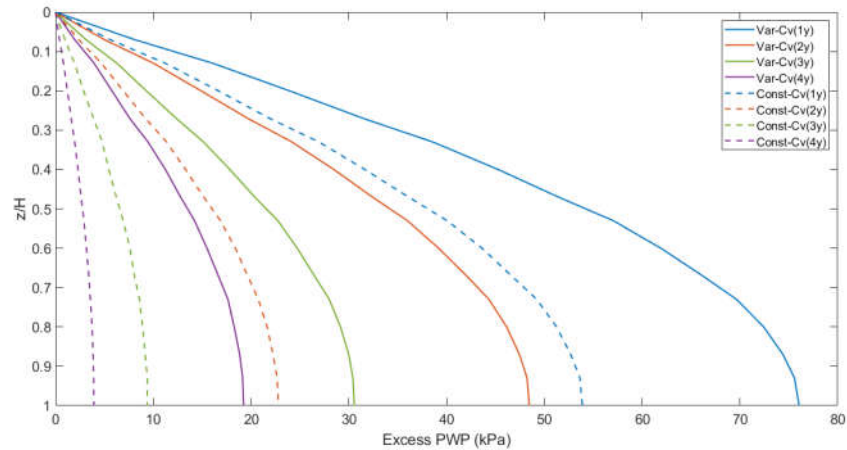


Figure 5. Results comparison for case 2

#### 4. Conclusion

This study presented a nonlinear solution for the one-dimensional consolidation equation incorporating spatial and temporal variations of the coefficient of consolidation. The Rationalized Haar Wavelet Transform was used to efficiently solve the resulting nonlinear differential equations. Based on the numerical analyses, the following conclusions are drawn:

- Excess pore water pressures obtained from the nonlinear solution with variable ( $C_v$ ) are consistently higher than those predicted by the classical constant parameter solution.
- The time required for full consolidation is significantly longer when the variability of ( $C_v$ ) is considered, indicating that Terzaghi's theory may underestimate consolidation duration in practical applications.
- The RHWT-based approach is flexible and capable of handling various functional forms of ( $C_v$ ), making it suitable for a wide range of nonlinear consolidation problems.

Future Research Directions:

- Extension of the method to two- and three-dimensional consolidation problems.
- Incorporation of large-strain effects and non-Darcian flow behavior.
- Coupled hydro-mechanical modeling of soft clays under cyclic or staged loading.
- Integration with machine-learning models to predict variable soil parameters.
- Experimental validation using laboratory consolidation tests with real-time monitoring of permeability

#### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding this article.

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